Integral Equation Model for Electromagnetic Scattering from a Rough Surface

Siddhant Gautam

Electrical Engineering,
Indian Institute of Technology Madras

July 23, 2018

Overview

Forward Model

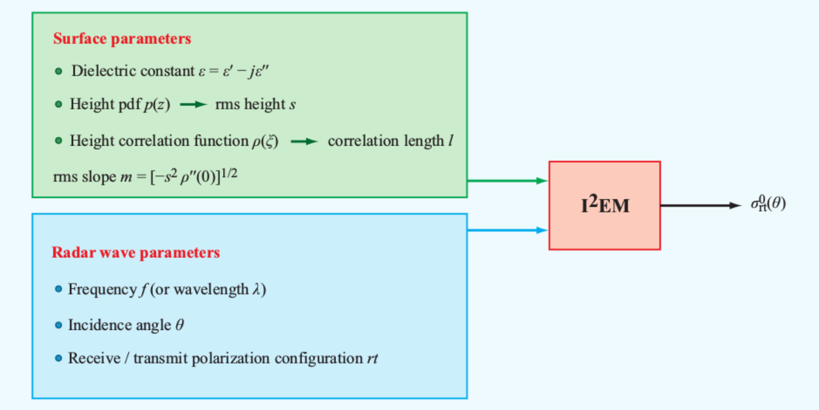


Figure: Improved Integral Equation Model (I^2EM)

[1]

[1] credits:Ulaby[2006]

Scattering from a rough surface

Incident Field

$$\vec{E}^i = \hat{a} E_0 exp(-jk_1 \hat{n}_i \cdot r) \quad (1)$$

[2] F. T. Ulaby, R. K. Moore and A. K. Fung, Microwave Remote Sensing, vol. 2, chapter 12. Reading, MA Addison-Wesley, 1982.

Scattering from a rough surface

Incident Field

$$\vec{E}^i = \hat{a}E_0 exp(-jk_1\hat{n}_i \cdot r) \quad (1)$$

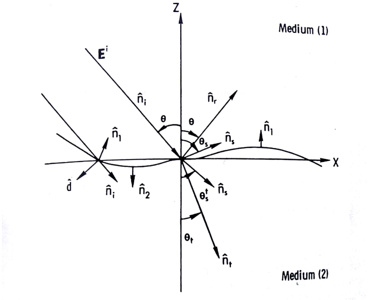


Figure: Geometry of surface scattering problem[2]

[2] F. T. Ulaby, R. K. Moore and A. K. Fung, Microwave Remote Sensing, vol. 2, chapter 12, Reading, MA Addison-Wesley, 1982.

Standard Integral Equations

Vector Wave Equation for Electric field

$$\nabla \times \nabla \times \vec{E} - k^2 \vec{E} = -j\omega\mu \vec{J} - \nabla \times \vec{K} \tag{2}$$

Standard Integral Equations

Vector Wave Equation for Electric field

$$\nabla \times \nabla \times \vec{E} - k^2 \vec{E} = -j\omega\mu \vec{J} - \nabla \times \vec{K} \tag{2}$$

Vector Greens Theorem

$$\int_V (\vec{Q} \cdot \nabla \times \nabla \times \vec{P} - \vec{P} \cdot \nabla \times \nabla \times \vec{Q}) dv = \int_\Sigma \vec{P} \times \nabla \times \vec{Q} - \vec{Q} \times \nabla \times \vec{P}) \cdot ds \tag{3}$$

Standard Integral Equations

Vector Wave Equation for Electric field

$$\nabla \times \nabla \times \vec{E} - k^2 \vec{E} = -j\omega\mu \vec{J} - \nabla \times \vec{K} \tag{2}$$

Vector Greens Theorem

$$\int_V (\vec{Q} \cdot \nabla \times \nabla \times \vec{P} - \vec{P} \cdot \nabla \times \nabla \times \vec{Q}) dv = \int_\Sigma \vec{P} \times \nabla \times \vec{Q} - \vec{Q} \times \nabla \times \vec{P}) \cdot ds \tag{3}$$

Substituting $\vec{P} = \vec{E}$ and $\vec{Q} = \phi \hat{p}$

$$\int_V j\omega\mu \vec{J}\phi + \vec{K} \times \nabla\phi - (\rho/\epsilon)\nabla\phi dv =$$

$$\int_\Sigma j\omega\mu(\hat{n}_s \times \vec{H})\phi - (\hat{n}_s \times \vec{E}) \times \nabla\phi - (\hat{n}_s \cdot \vec{E})\nabla\phi ds \tag{4}$$

Standard Integral Equations

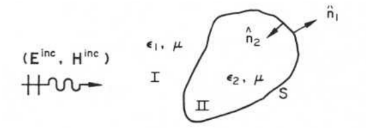


Figure: Scattering from a dielectric body

Standard Integral Equations

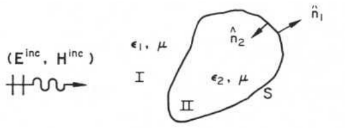


Figure: Scattering from a dielectric body

When source recedes to infinity and the volume currents are zero

$$\vec{E}(x) = \vec{E}^i - \frac{1}{4\pi}\oint_S j\omega\mu(\hat{n}' \times \vec{H})\phi - (\hat{n}' \times \vec{E}) \times \nabla'\phi - (\hat{n}' \cdot \vec{E})\nabla'\phi ds' \quad (5)$$

Standard Integral Equations[3]

Surface Tangential Fields

$$\hat{n} \times \vec{E} = 2\hat{n} \times \vec{E}^i - \frac{2}{4\pi}\hat{n} \times \int \vec{\mathcal{E}} ds' \tag{6}$$

$$\hat{n} \times \vec{H} = 2\hat{n} \times \vec{H}^i + \frac{2}{4\pi}\hat{n} \times \int \vec{\mathcal{H}} ds' \tag{7}$$

[3] Poggio, A. J., and E. K. Miller, "Integral equation solution of three dimensional scattering problems," Computer Techniques for Electromagnetics, Chapter 4, Pergmon, New York, 1973.

Standard Integral Equations[3]

Surface Tangential Fields

$$\hat{n} \times \vec{E} = 2\hat{n} \times \vec{E}^i - \frac{2}{4\pi}\hat{n} \times \int \vec{\mathcal{E}} ds' \tag{6}$$

$$\hat{n} \times \vec{H} = 2\hat{n} \times \vec{H}^i + \frac{2}{4\pi}\hat{n} \times \int \vec{\mathcal{H}} ds' \tag{7}$$

where

$$\vec{\mathcal{E}} = jk_1\eta_1(\hat{n}' \times \vec{H})G_1 - (\hat{n}' \times \vec{E_1}) \times \nabla' G - (\hat{n}' \cdot \vec{E})\nabla' G \tag{8}$$

$$\vec{\mathcal{H}} = jk/\eta_1(\hat{n}' \times \vec{E})G_1 + (\hat{n}' \times \vec{H}) \times \nabla' G + (\hat{n}' \cdot \vec{H})\nabla' G \tag{9}$$

[3] Poggio, A. J., and E. K. Miller, "Integral equation solution of three dimensional scattering problems," Computer Techniques for Electromagnetics, Chapter 4, Pergmon, New York, 1973.

Surface Tangential fields

A pair of integral equations

$$\hat{n} \times \vec{E} = (\hat{n} \times \vec{E})_k + (\hat{n} \times \vec{E})_c \tag{10}$$

$$\hat{n} \times \vec{H} = (\hat{n} \times \vec{H})_k + (\hat{n} \times \vec{H})_c \tag{11}$$

Surface Tangential fields

A pair of integral equations

$$\hat{n} \times \vec{E} = (\hat{n} \times \vec{E})_k + (\hat{n} \times \vec{E})_c \quad (10)$$

$$\hat{n} \times \vec{H} = (\hat{n} \times \vec{H})_k + (\hat{n} \times \vec{H})_c \quad (11)$$

Fields under Kirchoff Approximation

$$(\hat{n}_1 \times \vec{E})_k = (\hat{n} \times \vec{E}^i)_k + (\hat{n} \times \vec{E}^r)_k \quad (12)$$

$$(\hat{n} \times \vec{H})_k = (\hat{n} \times \vec{H}^i)_k + (\hat{n} \times \vec{H}^i)_k \quad (13)$$

Surface Tangential fields

A pair of integral equations

$$\hat{n} \times \vec{E} = (\hat{n} \times \vec{E})_k + (\hat{n} \times \vec{E})_c \tag{10}$$

$$\hat{n} \times \vec{H} = (\hat{n} \times \vec{H})_k + (\hat{n} \times \vec{H})_c \tag{11}$$

Fields under Kirchoff Approximation

$$(\hat{n}_1 \times \vec{E})_k = (\hat{n} \times \vec{E}^i)_k + (\hat{n} \times \vec{E}^r)_k \tag{12}$$

$$(\hat{n} \times \vec{H})_k = (\hat{n} \times \vec{H}^i)_k + (\hat{n} \times \vec{H}^i)_k \tag{13}$$

Complementary surface fields are defined as

$$(\hat{n} \times \vec{E})_c = \hat{n} \times (\vec{E}^i - \vec{E}^r) - \frac{2}{4\pi}\hat{n}_1 \times \int \vec{\mathcal{E}} ds' \tag{14}$$

$$(\hat{n} \times \vec{H})_c = \hat{n} \times (\vec{H}^i - \vec{H}^r) + \frac{2}{4\pi}\hat{n} \times \int \vec{\mathcal{H}} ds' \tag{15}$$

Estimation of Tangential Surface Fields

Vertical Polarization

$$(\hat{n} \times \vec{E})_{kv} \approx (1 - R_{\|})\hat{n} \times \hat{v}E^{i} \tag{16}$$

Estimation of Tangential Surface Fields

Vertical Polarization

$$(\hat{n} \times \vec{E})_{kv} \approx (1 - R_{\parallel})\hat{n} \times \hat{v} E^{i} \tag{16}$$

Horizontal Polarization

$$(\hat{n} \times \vec{E})_{kh} \approx (1 + R_{\perp})\hat{n} \times \hat{h} E^{i} \tag{17}$$

Estimation of Tangential Surface Fields

Vertical Polarization

$$(\hat{n} \times \vec{E})_{kv} \approx (1 - R_{\parallel})\hat{n} \times \hat{v}E^{i} \tag{16}$$

Horizontal Polarization

$$(\hat{n} \times \vec{E})_{kh} \approx (1 + R_{\perp})\hat{n} \times \hat{h}E^{i} \tag{17}$$

Cross Polarization

$$(\hat{n} \times \vec{E})_{kh} \approx (1 - (R_{\perp} + R_{\parallel})/2)(\hat{n} \times \hat{p})E^{i} \tag{18}$$

Far zone backscattered field

Stratton-Chu Integral

$$E^{s}_{qp} = -\frac{jk}{r\pi R} exp(-jkR) \int \hat{q} \cdot [-\hat{k}_i \times (\hat{n} \times \vec{E}_p) + \eta(\hat{n} \times \vec{H}_p)] \quad (19)$$

[4] where

$$\vec{k_i} = k\vec{k_i} = k(\hat{x}sin\theta - \hat{z}cos\theta) = \hat{x}k_x - \hat{z}k_z \quad (20)$$

[4] Stratton, J.A., Electromagnetic Theory, McGraw-Hill, NY, 1941

Far zone backscattered field

Stratton-Chu Integral

$$E_{qp}^{s} = -\frac{jk}{r\pi R} exp(-jkR) \int \hat{q} \cdot [-\hat{k}_i \times (\hat{n} \times \vec{E}_p) + \eta(\hat{n} \times \vec{H}_p)] \qquad (19)$$

[4] where

$$\vec{k}_i = k\vec{k}_i = k(\hat{x}sin\theta - \hat{z}cos\theta) = \hat{x}k_x - \hat{z}k_z \qquad (20)$$

Polarisation vectors $\hat{q}$ and $\hat{p}$ may be equal to $\hat{h}$ or $\hat{v}$

$$\hat{h} = \hat{y} \qquad (21)$$

$$\hat{v} = \hat{x}cos\theta + \hat{z}sin\theta \qquad (22)$$

[4] Stratton, J.A., Electromagnetic Theory, McGraw-Hill, NY, 1941

Far zone backscattered field

Kirchoff and Complementary Components of scattered field

$$E_{qp}^{k} = CE_0 \int f_{qp} exp(-2j\vec{k_i} \cdot \vec{r}) dx\ dy \tag{23}$$

$$E_{qp}^{c} = \frac{CE_0}{8\pi^2} \int F_{qp} exp[ju(x - x') + jv(y - y') - j\hat{k_i} \cdot \vec{r} - j\vec{k_i} \cdot \vec{r'}] dx\ dy\ dx'\ dy'\ du\ dv \tag{24}$$

Far zone backscattered field

Kirchoff and Complementary Components of scattered field

$$E^k_{qp} = CE_0 \int f_{qp} exp(-2j\vec{k_i} \cdot \vec{r}) dx\ dy \tag{23}$$

$$E^c_{qp} = \frac{CE_0}{8\pi^2} \int F_{qp} exp[ju(x - x') + jv(y - y') - j\hat{k_i} \cdot \vec{r} - j\vec{k_i} \cdot \vec{r'}] dx\ dy\ dx'\ dy'\ du\ dv \tag{24}$$

Field Coefficients

$$f_{qp} = \left[\hat{k_i} \times \hat{q} \cdot (\hat{n} \times \vec{E})_{kp} + \eta\hat{q} \cdot (\hat{n} \times \vec{H})_{kp}\right] \cdot (D_1/E^i) \tag{25}$$

$$F_{vv} = 8\pi^2 \left[\hat{k_i} \times \hat{q} \cdot (\hat{n} \times \vec{E})_{cp} + \eta\hat{q} \cdot (\hat{n} \times \vec{H})_{cp}\right] \cdot (D_1/E^i) \tag{26}$$

where $D_1 = \sqrt{Z_x^2 + Z_y^2 + 1}$

Power Calculation

Incoherent power = total power - subtract mean squared power

$$\langle E^s_{qp} E^{s*}_{qp}\rangle - \langle E^s_{qp}\rangle\langle E^s_{qp}\rangle^* = \langle E^k_{qp} E^{k*}_{qp}\rangle - \langle E^k_{qp}\rangle\langle E^k_{qp}\rangle^* \\ + 2Re\left[\langle E^c_{qp} E^{k*}_{qp}\rangle - \langle E^c_{qp}\rangle\langle E^k_{qp}\rangle^*\right] \\ + \langle E^c_{qp} E^{c*}_{qp}\rangle - \langle E^c_{qp}\rangle\langle E^c_{qp}\rangle^* \quad (27)$$

Power Calculation

Incoherent power = total power - subtract mean squared power

$$\langle E_{qp}^{s} E_{qp}^{s*}\rangle - \langle E_{qp}^{s}\rangle\langle E_{qp}^{s}\rangle^{*} = \langle E_{qp}^{k} E_{qp}^{k*}\rangle - \langle E_{qp}^{k}\rangle\langle E_{qp}^{k}\rangle^{*} + 2Re\left[\langle E_{qp}^{c} E_{qp}^{k*}\rangle - \langle E_{qp}^{c}\rangle\langle E_{qp}^{k}\rangle^{*}\right] + \langle E_{qp}^{c} E_{qp}^{c*}\rangle - \langle E_{qp}^{c}\rangle\langle E_{qp}^{c}\rangle^{*} \quad (27)$$

Kirchhoff Power term (assuming Gaussain height distribution)

$$P_{qp}^{k} = |CE_0 f_{qp}|^2 A_0 exp(-4k_z^2\sigma^2) \cdot \int \left[exp[4k_z^2\sigma^2\rho(\xi,\zeta)] - 1\right] e^{-2jk_x\xi} d\xi \, d\zeta \quad (28)$$

where $\zeta = x - x'$,$\xi = y - y'$ and A_0 is the illuminated area, σ^2 is the variance of surface and $\rho(\xi,\zeta)$ is the surface correlation.

Backscattering Coefficient

$$\sigma_{qp}^{0} = \sigma_{qp}^{k} + \sigma_{qp}^{kc} + \sigma_{qp}^{c} = (4\pi R^2 P_{qp})/(E_0^2 A_0) \tag{29}$$

Backscattering Coefficient

$$\sigma^0_{qp} = \sigma^k_{qp} + \sigma^{kc}_{qp} + \sigma^c_{qp} = (4\pi R^2 P_{qp})/(E_0^2 A_0) \tag{29}$$

Kirchhoff Backscatter term

$$\sigma^k_{qp} = \frac{k^2}{4\pi}\left[|f_{qp}|^2 exp(-4k_z^2\sigma^2)\right] \cdot \int \left[exp[4k_z^2\sigma^2\rho(\xi,\zeta)] - 1\right] e^{-2jk_x\xi} d\xi\ d\zeta \tag{30}$$

Backscattering Coefficient

$$\sigma^0_{qp} = \sigma^k_{qp} + \sigma^{kc}_{qp} + \sigma^c_{qp} = (4\pi R^2 P_{qp})/(E_0^2 A_0) \tag{29}$$

Kirchhoff Backscatter term

$$\sigma^k_{qp} = \frac{k^2}{4\pi}\left[|f_{qp}|^2 exp(-4k_z^2\sigma^2)\right] \cdot \int \left[exp[4k_z^2\sigma^2\rho(\xi,\zeta)] - 1\right] e^{-2jk_x\xi} d\xi \ d\zeta \tag{30}$$

Spectral Representation of Greens Function

$$G = \left(\frac{1}{2\pi}\right) \int \frac{j}{q} exp[(-jq)|z - z'| + ju(x - x') + jv(y - y')] du \ dv] \tag{31}$$

Soil Profile

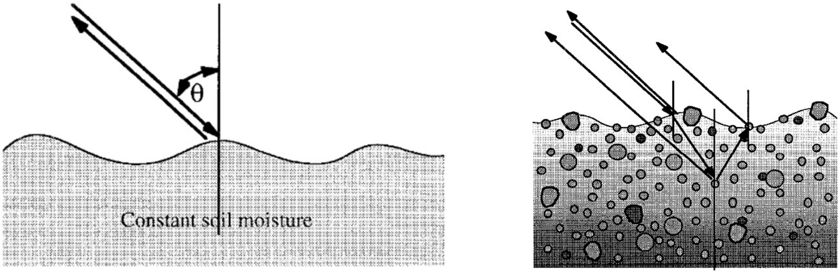


Figure: Homogeneous and Heterogeneous Soil Profile[5]

[5] A. K. Fung et al., "A modified IEM model for: scattering from soil surfaces with application to soil moisture sensing," Geoscience and Remote Sensing Symposium, 1996

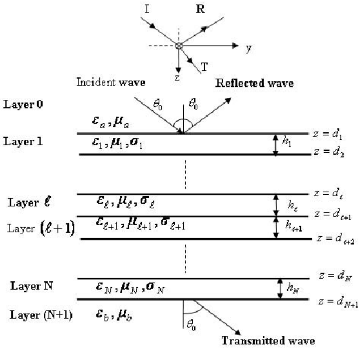


Figure: Multilayer dielectric structure[6]

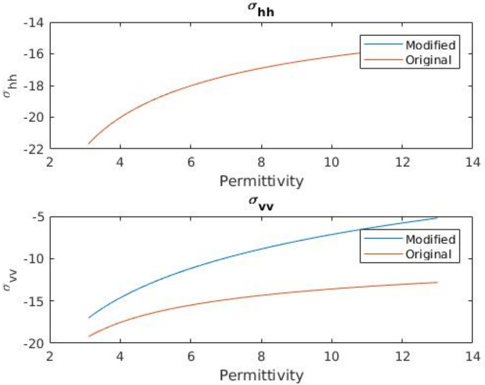
$$R_{ij} = \frac{n_i cos\theta_i - n_j cos\theta_j}{n_i cos\theta_i + n_j cos\theta_j} \quad (32) \qquad T_{ij} = \frac{2n_i cos\theta_i}{n_i cos\theta_i + n_j cos\theta_j} \quad (33)$$

[6] Analysis of planar dielectric multilayers as fss by Transmission Line Transfer Matrix Method (TLTMM)

Two layer IEM

Fresnel Reflection Coefficient

$$R_{\perp} = T_{12}R_{23}\left(\frac{T_{21}e^{-2jk_2d}}{1 - R_{21}R_{23}e^{-2jk_2d}}\right) \tag{34}$$



Modified IEM

Transitional dielectric layer

$$\epsilon_r(z) = 1 + \epsilon_{r_\infty} \frac{exp(mz)}{1 + exp(mz)} \tag{35}$$

Modified IEM

Transitional dielectric layer

$$\epsilon_r(z) = 1 + \epsilon_{r_\infty} \frac{exp(mz)}{1 + exp(mz)} \tag{35}$$

Fresnel Coefficient for Perpendicular Polarization

$$R_\perp = \frac{\Gamma(jScos\theta)}{\Gamma(-jScos\theta)} \frac{\Gamma\left[(-jS/2)(cos\theta + \sqrt{cos^2\theta + \epsilon_{r_\infty} - 1})\right]}{\Gamma\left[(jS/2)(cos\theta - \sqrt{cos^2\theta + \epsilon_{r_\infty} - 1})\right]} \times \frac{\Gamma\left[1 - (jS/2)(cos\theta + \sqrt{cos^2\theta + \epsilon_{r_\infty} - 1})\right]}{\Gamma\left[1 + (jS/2)(cos\theta - \sqrt{cos^2\theta + \epsilon_{r_\infty} - 1})\right]} \tag{36}$$

where $S = 2k/m$ and m is the transition rate factor

Modified IEM

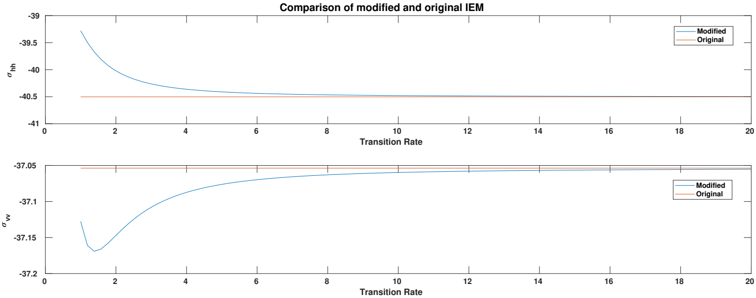
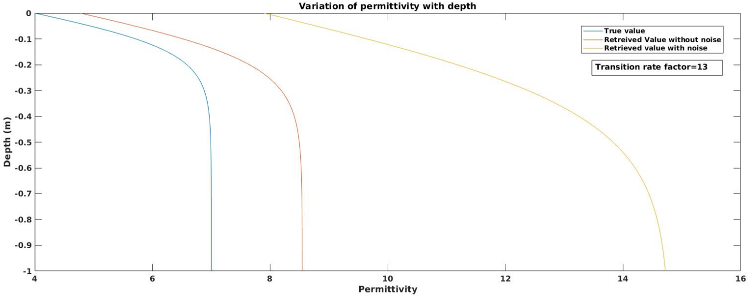
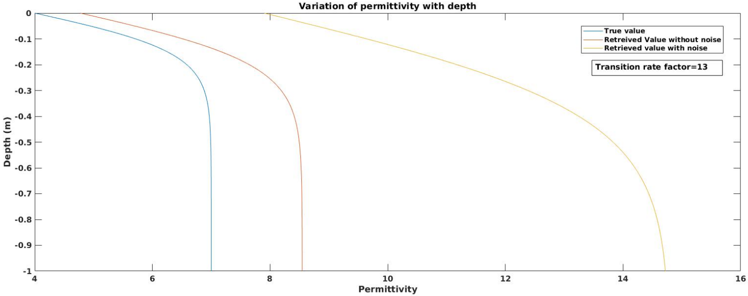


Figure: Backscattering Coefficients calculated from standard and modified IEM

Retrieved Permittivity



Retrieved Permittivity



RMS Errors:

Type	h	l	ϵ	m
Validation	0.068	5.23	1.79	1.79
Test data	0.1	5.73	2.92	3.27

Future work

1. Make a modified forward model in which multilayer formulation for moisture can be applied
2. Current model is only for bare soil. Suitable model has to be built that takes into account vegetation such as tree, canopy, etc

References

- Ulaby, Fawwaz Tayssir, and David G. Long, "Microwave Radar and Radiometric Remote Sensing," University of Michigan Press, 2014.
- K.-L. Chen, K.-S. Chen, Z.-L. Li, and Y. Liu, "Extension and Validation of an Advanced Integral Equation Model for Bistatic Scattering from Rough Surfaces," Progress In Electromagnetics Research, Vol. 152, 59-76, 2015.
- N. Baghdadi, J. Abou Chaaya and M. Zribi, "Semiempirical Calibration of the Integral Equation Model for SAR Data in C-Band and Cross Polarization Using Radar Images and Field Measurements," in IEEE Geoscience and Remote Sensing Letters, vol. 8, no. 1, pp. 14-18, Jan. 2011.
- A. K. Fung, Z. Li and K. S. Chen, "Backscattering from a randomly rough dielectric surface," in IEEE Transactions on Geoscience and Remote Sensing, vol. 30, no. 2, pp. 356-369, Mar 1992.

Thank You