Convolutional Neural Network for MRI Reconstruction Problem[1]

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[1] Jin, K.H., McCann, M.T., Froustey, E. and Unser, M., 2017. Deep Convolutional Neural Network for Inverse Problems in Imaging. IEEE Transactions on Image Processing, 26(9), pp.4509-4522.

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Introduction

Main goal: Using a *Convolutional Neural Network* based algorithm for solving ill-posed MRI reconstruction problem

- In an inverse problem, the measurement Y can be written as

$$Y = Ax + \eta$$

  where x is the image, A denotes the forward operator and η is the noise.

- **Inverse Reconstruction problem:**

$$\hat{x} = \arg\min_{x} \|Ax - y\|_2^2 + R(x)$$

  where $R(x)$ is a regularizer that assumes a prior on x.

- If $R(x) = 0$, the solution is given by $\hat{x} = A^{\dagger}y$ which minimises the $l2$-norm of the difference without assuming any prior on x.

Forward operator A

The forward or sensing operator A for different cases is given by:

- $A = I$ and $R(x) = ||x||_1$ for image denoising.
- A is subsampled Fourier transform, $R(x) = ||x||_{Tv}$ for MRI reconstruction problem.
- A is filtering for deblurring.

The direct inverse solution of the MRI reconstruction is

$$f = H^{\dagger} W g$$

where $g = Hf$ and g denotes the measurement, H is the Fourier transform matrix, and W is the inverse sparsifying transform.

Iterative Inversion

Inverse Reconstruction Problem

$$\hat{a} = \arg\min_a ||y - HWa||_2^2 + \lambda||a||_1$$

where H is the forward model (FT), y is the measured k-space data, a is transform coefficient vector and W is the sparsifying transform.
Desired reconstruction: $\boxed{\hat{x} = W\hat{a}}$

Iterative Shrinkage Thresholding Algorithm (ISTA)

$$a^{k+1} = S_{\frac{\lambda}{L}}(\frac{1}{L}W^*H^*y + (I - \frac{1}{L}W^*H^*HW)a^k)$$

where $S_{\frac{\lambda}{L}}$ is the soft-thresholding operator[2] and W^*, H^* are adjoint operators of W and H respectively.

[2] I. Daubechies et. al., "An iterative thresholding algorithm for linear inverse problems with a sparsity constraint," Commun. Pure Appl. Math., vol. 57, 2004.

Why use a CNN?

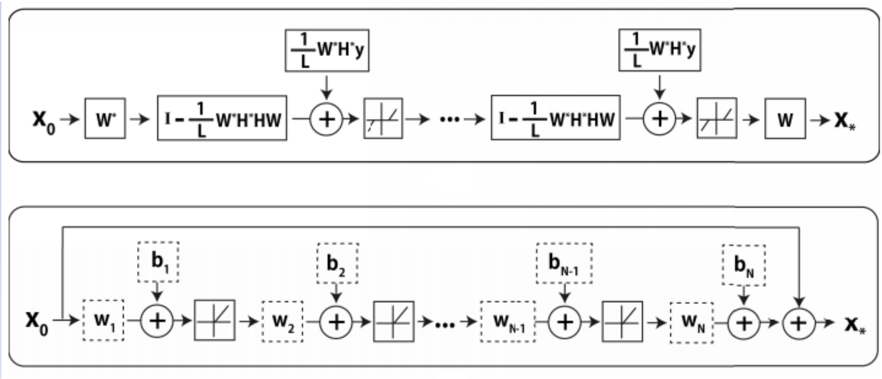
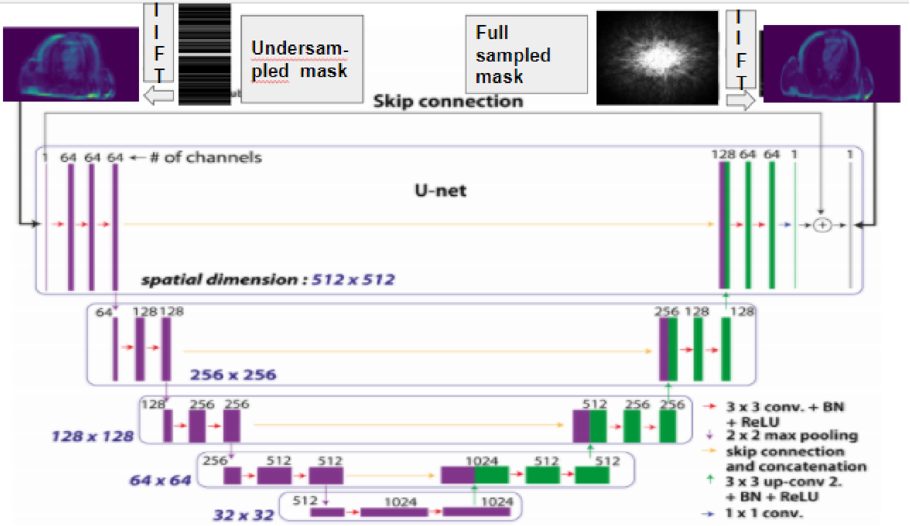


Figure: Iterative Unrolling of optimization problem modelled as a CNN[3]

[3] K. H. Jin, M. T. McCann, E. Froustey and M. Unser, "Deep Convolutional Neural Network for Inverse Problems in Imaging," in IEEE Transactions on Image Processing, vol. 26, Sept. 2017

Structure of U-Net[4]



[4]K. H. Jin et. al., "Deep Convolutional Neural Network for Inverse Problems in Imaging," in IEEE Transactions on Image Processing, vol. 26, Sept. 2017

Parts of U-Net

Components of U-Net:

1. **Double Convolution:** $2\times$ (Convolution + Batch-Normalization + ReLU)
2. **Downscaling:** Maxpooling by factor 2
3. **Upscaling:** Upsampling with bilinear interpolation
4. **Skip connection:** The input image is added to the output of the network. So that the network now learns the difference between the true and the aliased image.
5. **Concatenation:** At each subsampled layer, the output of downsample+Conv2d is concatenated to the upsampled image.

Advantage of U-Net architecture

- **Multilevel decomposition:** Extracts features from each scale
- **Multichannel filtering:** Extracts multiple feature maps from each scale

Masks Used:

1. Low Frequency (LF) - Vertical
2. Variable Density Random Sampling (VDRS)

Parameter	**Value**
Learning Rate	5×10^{-3}
Epochs	40
Batch-size	25
Training/Validation Ratio	645/50
Training/Testing Ratio	645/5
CNN Architecture	U-Net[5]

[5] Ronneberger, Olaf, Philipp Fischer, and Thomas Brox. "U-net: Convolutional networks for biomedical image segmentation." In International Conference on Medical image computing and computer-assisted intervention, pp. 234-241. Springer, Cham, 2015.

Implementation Details - Training Data Generation

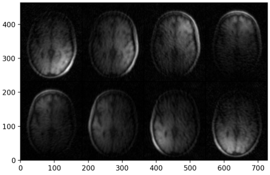
Parameter	Value
Data	OCMR[6]
Size of Image	512×208
No. of Total Samples	700
Undersampling Factors	8x, 16x, 24x
No. of Coils	18
Sensitivity Maps	ESPIRiT[7]
Decoder	CS-Sigpy[8]

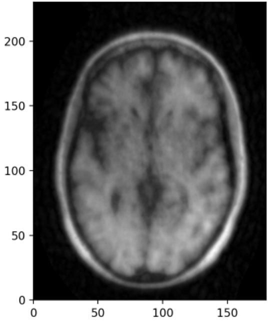
[6] Chen, Chong, et al. "OCMR (v1. 0)–Open-Access Multi-Coil k-Space Dataset for Cardiovascular Magnetic Resonance Imaging." arXiv e-prints (2020): arXiv-2008.

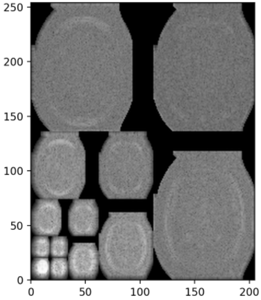
[7] Uecker, Martin, et.al. "ESPIRiT—An Eigenvalue Approach to Autocalibrating Parallel MRI: where SENSE meets GRAPPA." Magnetic Resonance in Medicine (2014).

[8] Ong F, Lustig M. SigPy: A Python Package for High Performance Iterative Reconstruction. ISMRM Proceedings 27th Annual Meeting, Montreal, May 2019

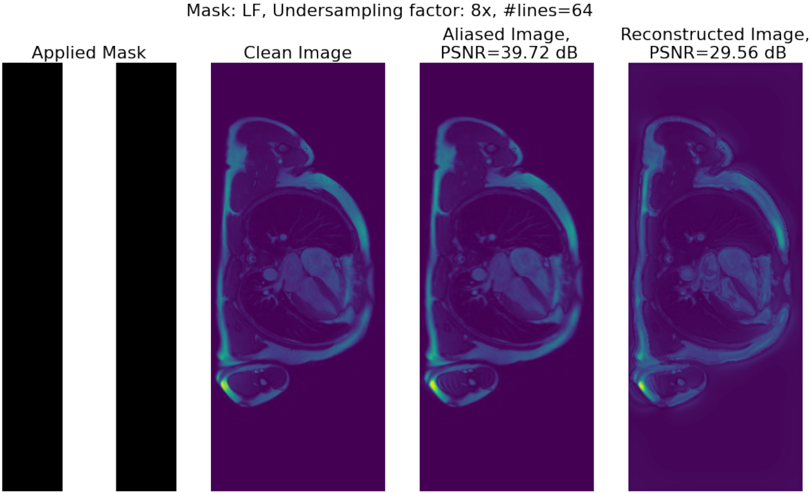
Training Data



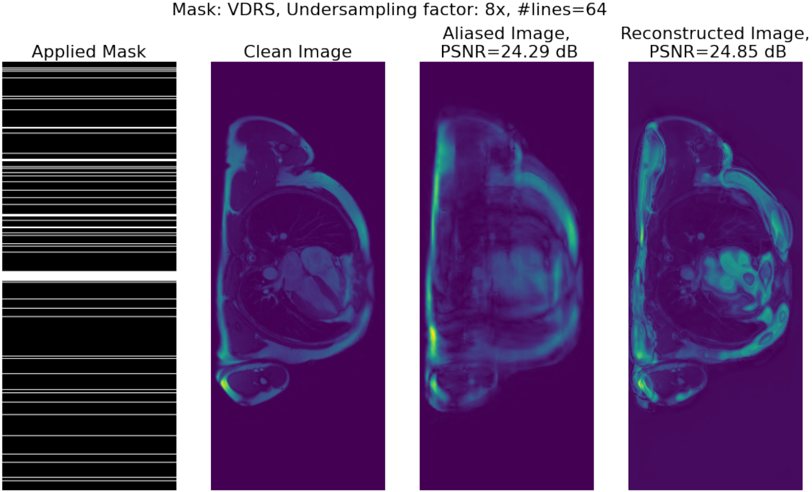




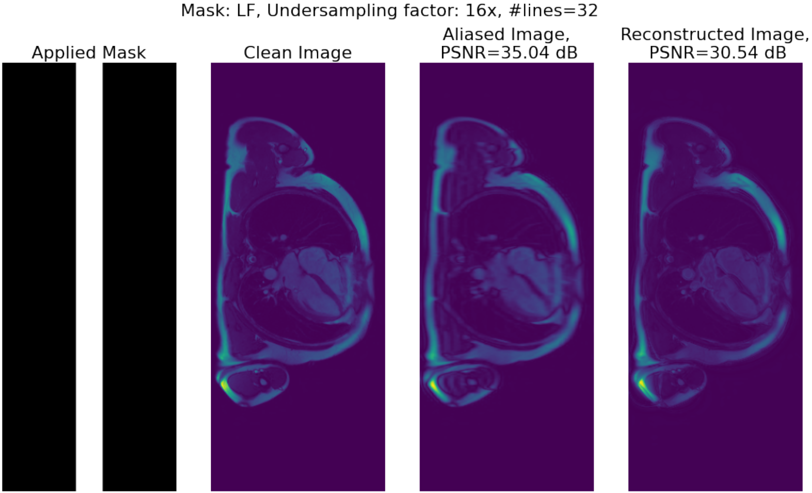
Reconstruction Results: Low Frequency @ 8x



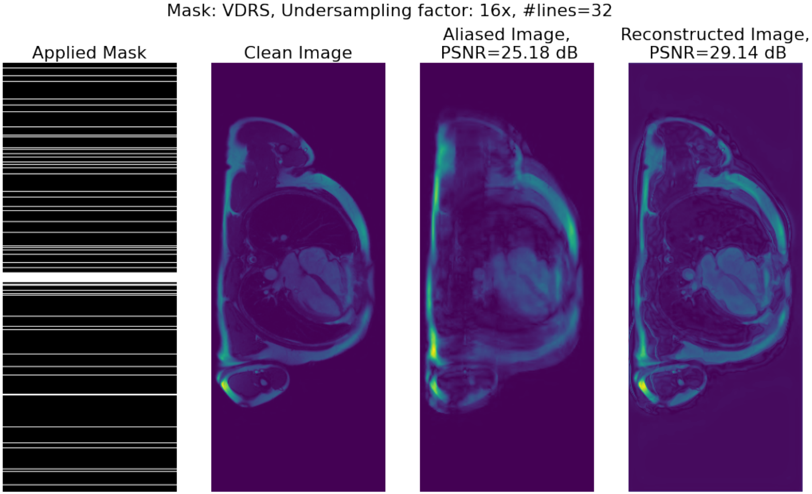
Reconstruction Results: Random Sampling @ 8x



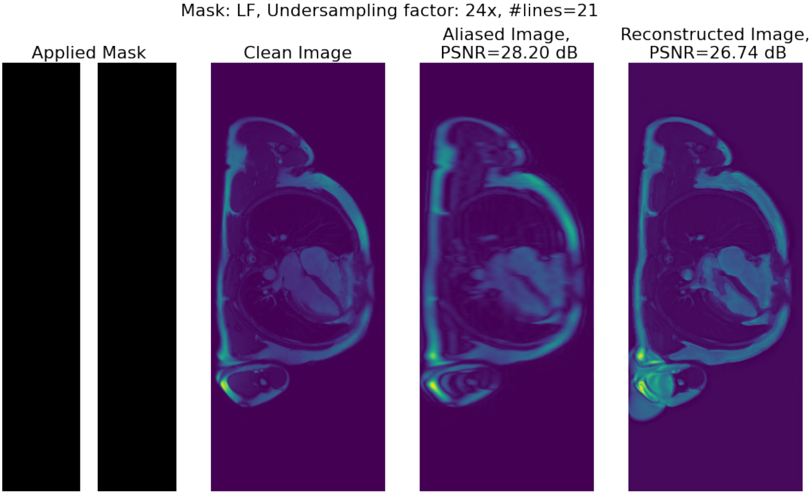
Reconstruction Results: Low Frequency @ 16x



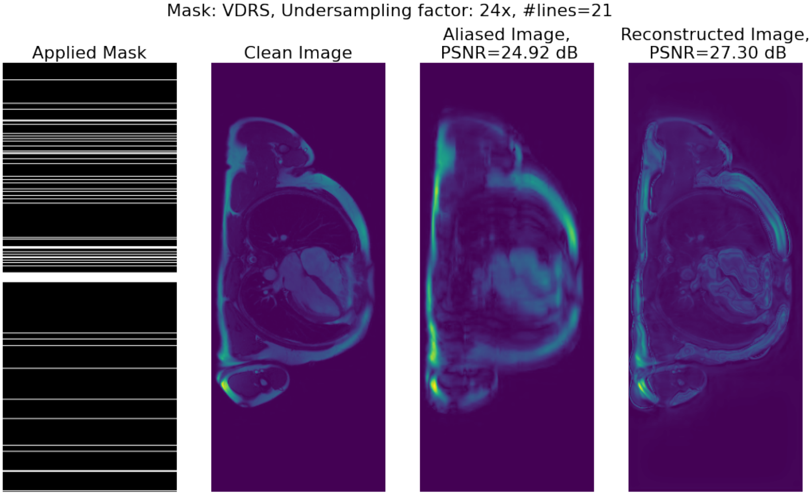
Reconstruction Results: Random Sampling @ 16x



Reconstruction Results: Low Frequency @ 24x



Reconstruction Results: Random Sampling @ 24x



Reconstruction Accuracy: PSNR

Mask Selected	Image	Undersampling Factor		
		8x	16x	24x
Low Frequency (Vertical)	Aliased	39.72	35.04	28.20
	Reconstructed	29.56	30.54	26.74
Random Sampling	Aliased	24.29	25.18	24.92
	Reconstructed	24.85	29.14	27.30

Table: PSNR (dB) of the aliased and reconstructed image from the U-Net for different cases

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Table: PSNR (dB) of the aliased and reconstructed image from the U-Net for different cases

- Better accuracy than the aliased image in case of variable density sampling
- No improvement in accuracy for 8x undersampling factor
- Significant improvement for highly undersampled MRI (16x, 24x)

Conclusion

- Presented a CNN based MRI-reconstruction with U-Net.
- U-Net extracts features of image at multiple scales. Hence it is ideal for MRI reconstruction.
- For the undersampled MRI dataset, U-Net can extract the fine detail in the image.
- For highly undersampled MRI with reduced time scan, better reconstruction with U-Net.

Fin.
Questions?